Exercises: Discrete-Time Convolution

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1. Graphical evaluation of convolution is used to find the (output, input) for a system given the (output, input) and the impulse response.

2. We use graphical evaluation of convolution to (develop insight regarding system behavior, obtain the system output faster than a computer).

3. Which of the following steps are involved in graphical evaluation of discrete-time convolution $y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$. Select all that apply.

a) Find $x[n-k]$ as a function of k for some fixed value n

b) Find $x[n-k]h[k]$ as a function of n for some fixed value k

c) Sum all the values in $x[n-k]h[k]$ over k for some fixed value n

d) Find $x[n-k]$ as a function of n for some fixed value k

e) Find $x[n-k]h[k]$ as a function of k for some fixed value n

f) Sum all the values in $x[n-k]h[k]$ over n for some fixed value k

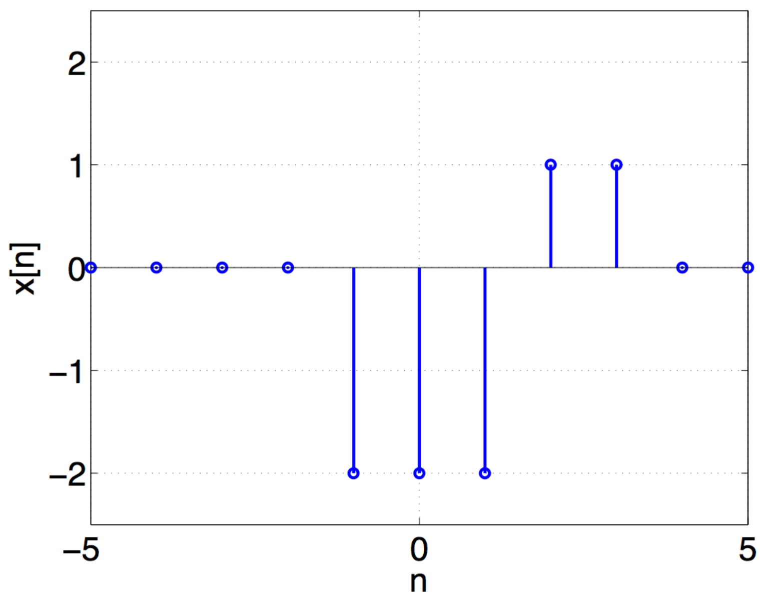
4. In order to find $x[n-k]$ for $n > 0$ you:

a) Reflect $x[k]$ about $k = 0$ to obtain $x[-k]$, then shift $x[-k]$ to the right for n steps

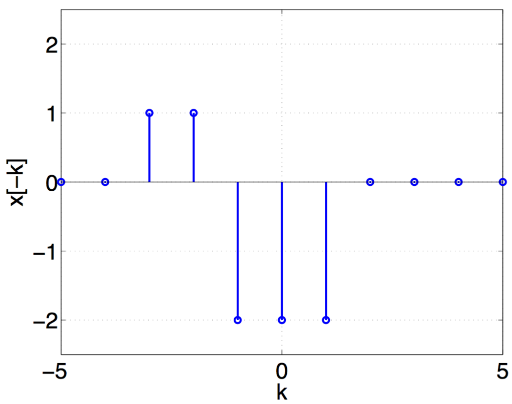
b) Reflect $x[k]$ about $k = 0$ to obtain $x[-k]$, then shift $x[-k]$ to the left for n steps

c) Shift $x[k]$ to the right for n steps, then reflect about $k = 0$

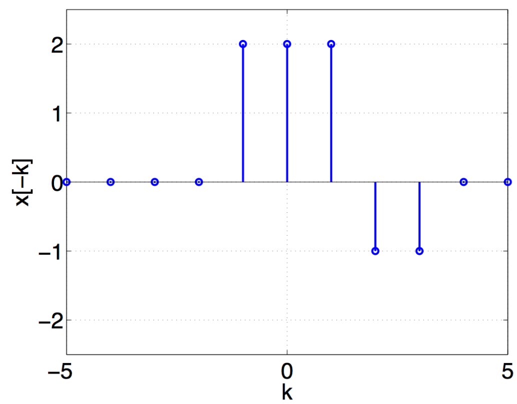
The discrete-time signal $x[n]$ is depicted in the figure below will be used in the next three problems.



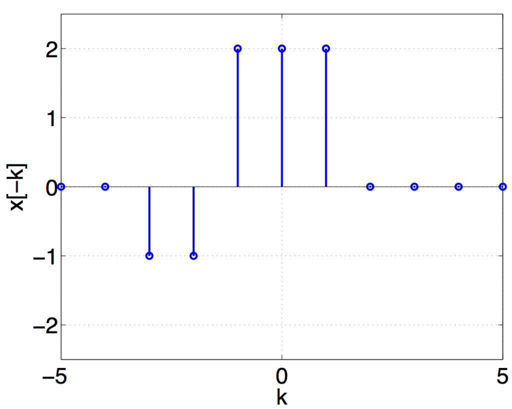
5. Select the answer corresponding to $x[-k]$.



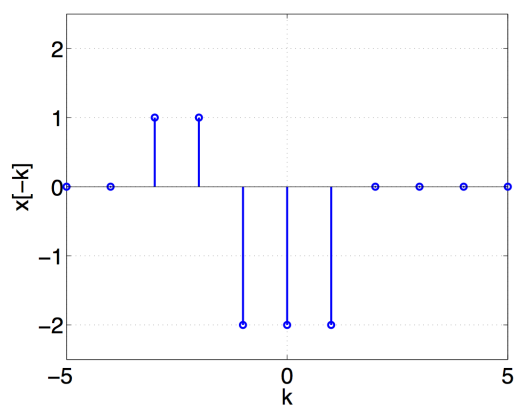
(a)



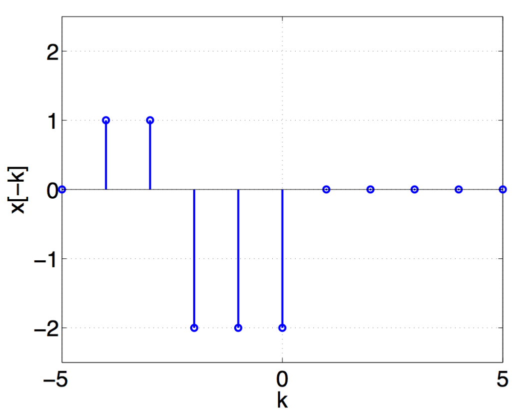
(b)



(c)

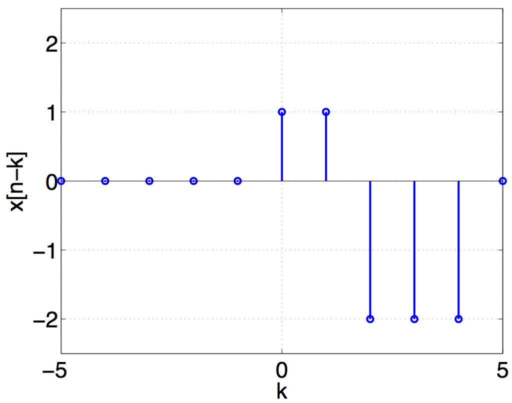


(d)

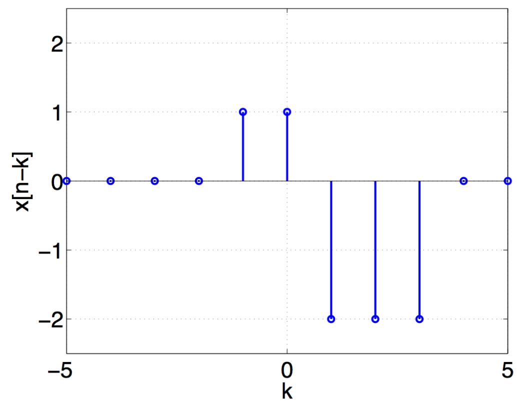


(e)

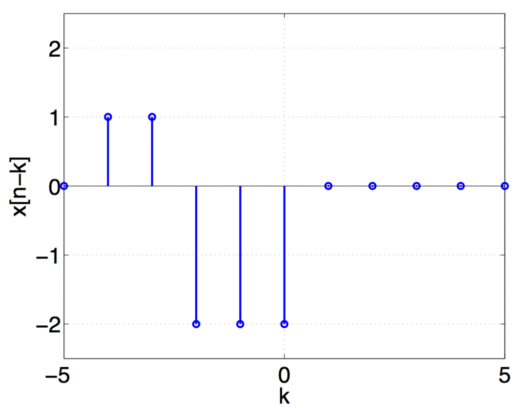
6. Each of the following graphs depict $x[n-k]$ for some value of n. Find the n corresponding to each graph.



(f)

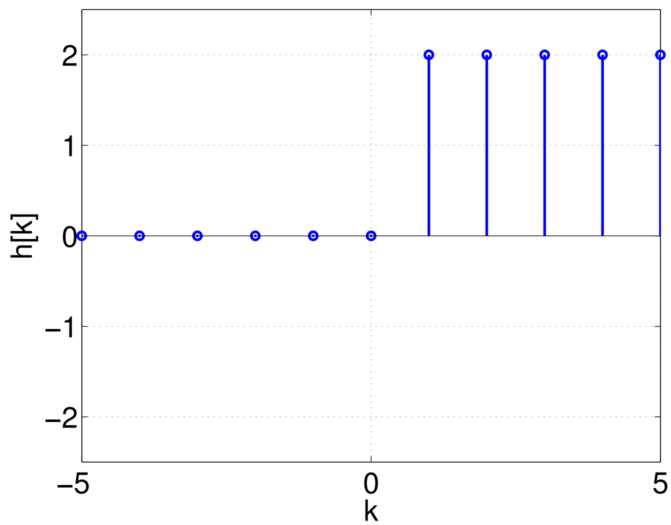


(g)

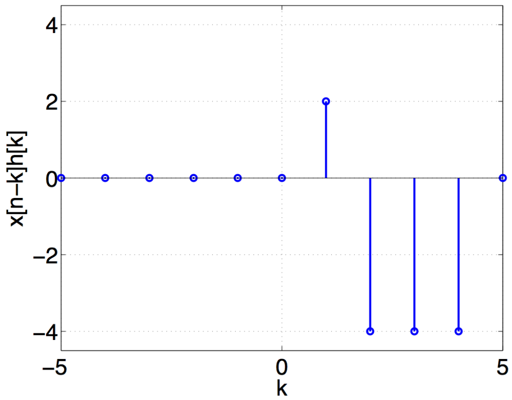


(h)

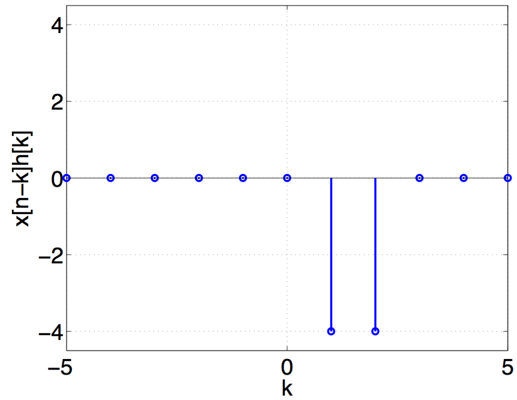
7. Find the output of a system $y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$ for $x[n]$ given previously. The impulse response of the system is $h[n] = 2u[n-1]$ as shown below.



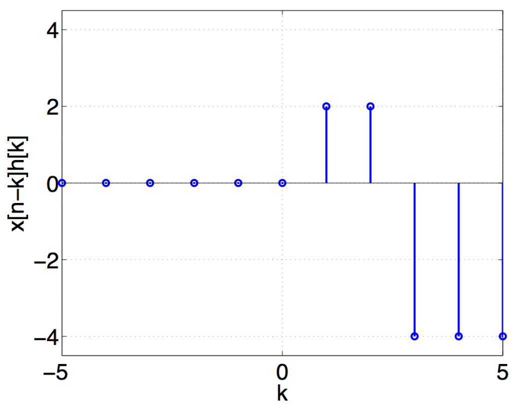
Each of the following graphs depict $x[n-k]h[k]$ for some value of n. Enter the n corresponding to each graph.



(i)



(j)



(k)

Now find the output $y[n]$ for the following values of n.

i) $y[-2]$

ii) $y[0]$

iii) $y[1]$

iv) $y[2]$

v) $y[3]$

vi) $y[10]$

vii) $y[100]$

8. In this problem we compute $y[n] = x[n] * h[n]$ where $h[n] = (3/4)^n u[n]$ and $x[n] = 2(u[n+2] - u[n-5])$.

a) Graph $x[k]$. Identify the value of $x[k]$ on the following intervals.

i) $x[k], k < -2$

ii) $x[k], -2 \leq k \leq 4$

iii) $x[k], k > 4$

b) Graph $x[-k]$. Identify the ranges of k for which $x[-k]$ takes the following values.

i) $x[-k] = 0$ for $k <$ _______

ii) $x[-k] = 2$ for _______ $\leq k \leq$ _______

iii) $x[-k] = 0$ for $k >$ _______

c) Graph $x[n-k]$ for $n = -5$. Enter the ranges of k for which $x[n-k]$ takes the following values:

i) $x[n-k] = 0$ for $k <$ _______

ii) $x[n-k] = 2$ for _______ $\leq k \leq$ _______

iii) $x[n-k] = 0$ for $k >$ _______

d) Graph $x[n-k]$ for $n = -1$. Enter the ranges of k for which $x[n-k]$ takes the following values:

i) $x[n-k] = 0$ for $k <$ _______

ii) $x[n-k] = 2$ for _______ $\leq k \leq$ _______

iii) $x[n-k] = 0$ for $k >$ _______

e) Graph $x[n-k]$ for $n = 1$. Enter the ranges of k for which $x[n-k]$ takes the following values:

i) $x[n-k] = 0$ for $k <$ _______

ii) $x[n-k] = 2$ for _______ $\leq k \leq$ _______

iii) $x[n-k] = 0$ for $k >$ _______

f) Graph $x[n-k]$ for $n = 3$. Enter the ranges of k for which $x[n-k]$ takes the following values:

i) $x[n-k] = 0$ for $k <$ _______

ii) $x[n-k] = 2$ for _______ $\leq k \leq$ _______

iii) $x[n-k] = 0$ for $k >$ _______

g) Graph $x[n-k]$ for $n = 6$. Enter the ranges of k for which $x[n-k]$ takes the following values:

i) $x[n-k] = 0$ for $k <$ _______

ii) $x[n-k] = 2$ for _______ $\leq k \leq$ _______

iii) $x[n-k] = 0$ for $k >$ _______

h) Graph $x[n-k]h[k]$ for $n = -1$. Enter the ranges of k for which $x[n-k]h[k]$ takes the following values:

i) $x[n-k]h[k] = 0$ for $k <$ _______

ii) $x[n-k]h[k] = 2(3/4)^k$ for _______ $\leq k \leq$ _______

iii) $x[n-k]h[k] = 0$ for $k >$ ______

i) Graph $x[n-k]h[k]$ for $n = 3$. Enter the ranges of k for which $x[n-k]h[k]$ takes the following values:

i) $x[n-k]h[k] = 0$ for $k <$ ______

ii) $x[n-k]h[k] = 2(3/4)^k$ for ______ $\leq k \leq$ ______

iii) $x[n-k]h[k] = 0$ for $k >$ ______

j) Graph $x[n-k]h[k]$ for $n = 6$. Enter the ranges of k for which $x[n-k]h[k]$ takes the following values:

i) $x[n-k]h[k] = 0$ for $k <$ ______

ii) $x[n-k]h[k] = 2(3/4)^k$ for ______ $\leq k \leq$ ______

iii) $x[n-k]h[k] = 0$ for $k >$ ______

k) Now find $y[n]$ for the following values of n.

Hint: you may find it helpful to use the formula for a finite geometric series

$$\sum_{k=K_0}^{K_1} a^k = a^{K_0}\left(\frac{1-a^{K_1-K_0+1}}{1-a}\right)$$

for $a = 3/4$. This formula applies for all $a \neq 0$. Note that if K_0 is close to K_1 then it is probably simpler to add up the sum rather than use this closed form expression.

i) $y[-1] =$ _______

ii) $y[3] =$ _______

iii) $y[6] =$ _______

l) The output is zero for $n <$ _______

m) As $n \to \infty$ the output $y[n] \to$ _______

n) As n increases from $n = -3$, the output initially (decreases, increases).